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3 (Sem-6/CBCS) STA HC 2

2023

**STATISTICS**

(Honours)

Paper : STA-HC-6026

**(Multivariate Analysis and  
Non-parametric Methods)**

Full Marks : 60

Time : Three hours

**The figures in the margin indicate  
full marks for the questions.**

1. Answer the following questions as directed :

1×7=7

(a) Write down the probability density function of the bivariate normal distribution.

(b) The characteristic function of the multivariate normal distribution having the mean vector  $\underline{Q}$  and variance covariance matrix  $I$ , is \_\_\_\_\_.

(Fill in the blank)

Contd.

(c) Non-parametric tests can be used only if the measurements are

- (i) nominal or ordinal
- (ii) ratio scale
- (iii) interval scale
- (iv) interval and ratio scale

(Choose the correct option)

(d) Statement : Non-parametric test does not make any assumption regarding the form of the population. The statement is

- (i) True
- (ii) False

(Tick the correct answer)

(e) If the correlation coefficient (e), is zero for a bivariate normal distribution, then the variables are

- (i) dependent
- (ii) independent
- (iii) uncorrelated but dependent
- (iv) partly dependent

(f) Let  $\underline{X}$  (with  $p$ -components) be

distributed according to  $N(\underline{\mu}, \Sigma)$ . Then

$\underline{Y} = C\underline{X}$  is distributed according to

(i)  $N(\underline{\mu}, C\Sigma C')$  for singular  $C$

(ii)  $N(C\underline{\mu}, \Sigma)$  for  $C$  non-singular

(iii)  $N(C\underline{\mu}, C\Sigma C')$  for  $C$  non-singular

(iv)  $N(O, I)$

(g) Define partial correlation coefficient.

2. Answer the following questions :  $2 \times 4 = 8$

(a) Explain the test for randomness.

(b) Find  $Cov(A\underline{X}, B\underline{Y})$  where  $A, B$  are matrices of constant elements.

(c) Let  $\underline{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ . If  $X_1$  and  $X_2$  are

independent and  $g(\underline{x}) = g^{(1)}(x_1)g^{(2)}(x_2)$ ,

prove that

$$E(g(X)) = E[g^{(1)}(X_1)] E[g^{(2)}(X_2)]$$

(d) Describe the advantages and drawbacks of the non-parametric methods over parametric methods.

3. Answer **any three** questions from the following : 5×3=15

(a) For a bivariate distribution

$$f(x, y) = \frac{1}{2\pi\sqrt{1-p^2}} \exp \left[ -\frac{1}{2(1-p^2)} (x^2 - 2pxy + y^2) \right],$$

$$-\infty < (x, y) < \infty.$$

Find the conditional distribution of  $Y$  given  $X$ .

(b) Let  $(X, Y)$  be a bivariate normal random variable with  $E(X) = E(Y) = 0$ ,  
 $V(X) = V(Y) = 1$  and  $Cov(X, Y) = p$ . Find the probability density function (pdf) of  $Z = Y/X$ .

(c) For a multivariate normal distribution

$$N(\underline{\mu}, \Sigma), \text{ if } \underline{\mu} = \underline{0} \text{ and}$$

$$\Sigma = \begin{pmatrix} 1 & 0.80 & -0.40 \\ 0.80 & 1 & -0.56 \\ -0.40 & -0.56 & 1 \end{pmatrix}$$

Find the partial correlation between  $X_1$  and  $X_3$  given  $X_2$ .

(d) Write a short note on principal component analysis.

(e) Describe the sign test for one sample.

4. Answer **either (a) or (b)** : 10

(a) Write a note on Hotelling  $T^2$  mentioning its applications. Prove that Hotelling  $T^2$  is invariant under a non-singular linear transformation.

$$4+6=10$$

**Or**

(b) For a bivariate normal distribution

$$dF = k \exp \left[ -\frac{2}{3} (x^2 - xy + y^2 - 3x + 3y + 3) \right] dx dy,$$

find —

- (i) the value of  $K$ ;
- (ii) marginal distribution of  $Y$ ;
- (iii) expectation of the conditional distribution of  $Y$  given  $X$ .

5. Answer **either** (a) **or** (b) : 10

(a) Describe explicitly the Kruskal-Wallis test with example.

**Or**

(b) Write a note on Wilcoxon-Mann-Whitney test for non-parametric methods.

6. Answer **either** (a) **or** (b) : 10

(a) Prove that if  $X_1, X_2, \dots, X_p$  have a joint normal distribution, a necessary and sufficient condition for one subset of the random variables and the subset consisting of the remaining variables to be independent is that each covariance of a variable from one set and a variable from the other set is 0 (zero).

**Or**

(b) Derive the pdf of bivariate normal distribution as a particular case of multivariate normal distribution.

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