2019

STATISTICS

(Major)

Paper : 4.2

(Descriptive Statistics—II and Probability—II)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×7=7

(a) If X is a random variable having probability function $f(x)$, then the function $\sum e^{itx} f(x)$, for i be an imaginary unit, is known as

(i) moment generating function

(ii) probability distribution function

(iii) characteristic function

(iv) None of the above

(Choose the correct option)

(b) The standard error of difference of two sample means, i.e., $(\bar{x}_1 - \bar{x}_2)$ is ______.

(Fill in the blank)

(c) State the 95% confidence limits for the population mean for large sample.

(d) Characteristic function of a random variable always exists.

(Write True or False)

(e) If X is a continuous random variable with mean μ and variance σ^2, then for any positive number k

$$P\{|X-\mu| \geq k\sigma\} \leq \frac{1}{k^2}$$

is known as

(i) Liapounoff's inequality

(ii) Markov's inequality

(iii) Chebyshev's inequality

(iv) None of the above

(Choose the correct option)

(f) Families of random variables which are functions of, say, time, are known as ______.

(Fill in the blank)

(g) Define null hypothesis.

2. Answer the following questions in short : 2×4=8

(a) State the conditions to be satisfied for weak law of large numbers to hold by a sequence of random variables $\{X_i\}$.

(b) Does there exist a variate X for which

$$P\{\mu_x - 2\sigma \leq X \leq \mu_x + 2\sigma\} = 0\cdot6?$$

(c) Define Markov chain and give an example.

(d) Define sampling distribution and standard error.

3. Answer any *three* of the following questions : 5×3=15

(a) Let $\{X_i\}$ be a sequence of independent random variables such that X_i assumes the values $\frac{1}{n}$ and $\frac{n+1}{n}$ with respective probabilities $\frac{1}{2n}$ and $\frac{2n-1}{2n}$. Examine whether the weak law of large numbers holds good.

(b) How large a sample must be taken in order that the probability will be at least 0·95 that $\overline{X}_n$ will be within 0·5 of μ (μ is unknown and $\sigma = 1$)?

(c) Suppose that the probability of a dry day (state 0) following a rainy day (state 1) is $\frac{1}{3}$ and that the probability of a rainy day following a dry day is $\frac{1}{2}$. State the corresponding transition probability matrix. Given that May 1 is a dry day, what is the probability that May 4 is a dry day?

(d) Describe the procedure to test the significance of difference between two proportions.

(e) Find the standard error of a linear function of a number of variables.

4. Answer the following questions : 10×3=30

(a) (i) Obtain unbiased estimate of population mean μ and variance σ^2. Also find the estimate of population variance for large sample. 3+4+3=10

Or

(ii) Find the expression for standard error of sample variance. Also show that

$$\text{cov}(\overline{X}, S^2) = \frac{\mu_3}{n}$$

notations having usual meaning. 7+3=10

(b) *(i)* If

$$X_i = \begin{cases} 1, & \text{with probability } p \\ 0, & \text{with probability } q \end{cases}$$

then prove that the distribution of the random variable

$$S_n = X_1 + X_2 + \cdots + X_n,$$

where X_i's are independent, is asymptotically normal as $n \to \infty$. 10

Or

(ii) Let $\overline{X}_n$ be the sample mean of a random sample of size n from rectangular distribution on [0, 1]. Let

$$U_n = \sqrt{n}\left(\overline{X}_n - \frac{1}{2}\right)$$

then show that

$$F(\mu) = \lim_{n \to \infty} P(U_n \leq \mu)$$

exists and determine it. 10

(c) *(i)* (1) Describe applications of stochastic processes in various fields other than mathematical application.

(2) Prove that the matrix given below is a transition probability matrix of an irreducible Markov chain :

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{pmatrix}$$

5+5=10

Or

(ii) (1) Write an explanatory note on the specification of stochastic processes.

(2) Let $\{X_n, n \geq 0\}$ be a Markov chain with three states 0, 1 and 2 with transition matrix

$$P = \begin{pmatrix} 1/3 & 2/3 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 1/2 & 1/2 \end{pmatrix}$$

Represent the above transition probability matrix with the help of a digraph. 6+4=10

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