2019

STATISTICS

(Major)

Paper : 2.1

(Numerical and Computational Techniques—I)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×7=7

(a) The value of $\Delta^3(4x^3)$, the interval of differencing being unity, is

(i) 0

(ii) 1

(iii) 24

(iv) 12

(Choose the correct option)

(b) The value of $\Delta^3 0^4$ is

(i) 1

(ii) 3

(iii) 41

(iv) None of the above

(Choose the correct option)

(c) The relationship between the operators μ, δ and D is

(i) $\mu = \frac{1}{2}(e^{-hD} - e^{hD})\delta$

(ii) $\mu\delta = \frac{1}{2}(e^{hD} - e^{-hD})$

(iii) $\mu\delta = \frac{1}{2}(e^{-hD} - e^{hD})$

(iv) None of the above

(Choose the correct option)

(d) Many difference equations can be solved by multiplying with some suitable function known as ______ and thus reducing the equation to integral form.

(Fill in the gap)

(e) Suppose the values of a function corresponding to the given values of the independent variable are known.

The process of evaluating the derivative(s) of the function at some particular value of the independent variable is known as

(i) numerical quadrature

(ii) numerical differentiation

(iii) inverse interpolation

(iv) None of the above

(Choose the correct option)

(f) Suppose $P(x)$ is a polynomial of degree 4 in x. Then the graph of $P(x)$ will cross the x-axis

(i) an odd number of times

(ii) an even number of times

(iii) 4 times

(iv) 2 times

(Choose the correct option)

(g) In Simpson's $\frac{3}{8}$th rule, the integrand is assumed to be a polynomial of

(i) 1st degree

(ii) 2nd degree

(iii) 3rd degree

(iv) None of the above

(Choose the correct option)

2. Answer the following questions : 2×4=8

(a) Prove that

$$1+\Delta=\sum_{j=0}^{\infty}\nabla^j$$

where Δ and ∇ are the forward and backward differences respectively.

(b) Obtain the function whose first difference is $3x^2+6x+5$.

(c) Solve the difference equation

$$u_{x+1}-5^x u_x=0$$

(d) If $u_x=a+bx+cx^2$, then prove that

$$\int_1^3 u_x dx=2u_2+\frac{1}{12}(u_0-2u_2+u_4)$$

3. Answer the following questions : 3×5=15

(a) Show that

$$(n+1)\Delta^n 0^n=2[\Delta^{n-1}0^n+\Delta^n 0^n]$$

(b) Solve :

$$u_{x+2}-4u_{x+1}+4u_x=2^x$$

(c) Using any interpolation formula, discuss how you will find the first and the second derivatives of a function numerically.

Or

Obtain the value of x when $y_x=19$, given the following values :

x	:	0	1	2
y_x	:	0	1	20

(d) With the help of Euler-Maclaurin summation formula, evaluate

$$1^p+2^p+\ldots+n^p$$

(e) State and prove Bessel's central difference formula.

4. Answer the following questions : 10×3=30

(a) Show by applying Lagrange's interpolation formula for arguments x_0, x_1 and $x_0+\varepsilon$ as $\varepsilon\to 0$, that $f(x)$ can be expressed in the form

$$f(x)=\frac{(x_1-x)(x+x_1-2x_0)}{(x_1-x_0)}f(x_0)$$
$$+\frac{(x-x_0)(x-x_1)}{(x_1-x_0)}f'(x_0)+\frac{(x-x_0)^2}{(x_1-x_0)^2}f(x_1)$$

Or

Solve the following difference equations :

(i) $u_{x+2} + a^2 u_x = \cos ax$

(ii) $u_{x+2} - 7u_{x+1} + 10u_x = 12 \cdot 5^x$

(b) If $f(x)$ is a second-degree polynomial in x and $u_{-1} = \int_{-3}^{-1} f(x)\,dx$, $u_0 = \int_{-1}^{1} f(x)\,dx$, $u_1 = \int_{1}^{3} f(x)\,dx$, then show that

$$f(0) = \frac{1}{2}\left[u_0 - \frac{\Delta^2 u_{-1}}{24}\right]$$

Or

State and prove Stirling's interpolation formula.

(c) Give the geometrical interpretation of Newton-Raphson method. Show that this method converges when

$$|f(x) f''(x)| < [f'(x)]^2$$

Or

From the following data, find $f'(10)$:

x	:	3	5	11	27	34
$f(x)$	:	−13	23	899	17315	35606

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