

2017

STATISTICS
(Major)

Paper : 3.1

(Mathematical Methods—II)

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Answer the following questions : 1×7=7

(a) Suppose A is a (4×5) matrix and $\rho(A) = 3$ (i.e., the rank of the matrix A). Let A_{44} be a fourth order minor of A . Find $\det(A_{44})$.

(b) Let A and B be two matrices such that

$$A = \begin{pmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{pmatrix} \text{ and } B = \begin{pmatrix} 9 & -1 & 6 & -3 \\ 36 & 1 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{pmatrix}$$

It is given that $\rho(A) = 4$. Find $\rho(B)$.

(2)

- (c) For the following system of homogeneous equations, i.e.,

$$(A)_{n \times n} (X)_{n \times 1} = (0)_{n \times 1}$$

it is given that $\rho(A) = n$. Find the number of linearly independent solutions that this system of equations will possess.

- (d) Suppose A is a (6×6) idempotent matrix given $\rho(I - A) = 4$ (I is a 6×6 identity matrix). Find $\rho(A)$.

- (e) Express the vector $\begin{pmatrix} 5 \\ 7 \\ 8 \end{pmatrix}$ as a linear

combination of the vectors

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ and } \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

- (f) Given that A is a (5×5) matrix and is non-singular. Find $\rho(A)$.
- (g) Consider a linear transformation which transforms the quadratic form $Q = X'AX$ into $Y'IY$, where $X = (X_1, X_2, \dots, X_n)^T$, A is an $n \times n$ matrix, $Y = (Y_1, Y_2, \dots, Y_n)^T$ and $I = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}_{n \times n}$, $r < n$ (i.e., I_r is an identity matrix of order r). Identify whether Q is a positive definite or positive semi-definite quadratic form.

(3)

2. Answer the following :

2×4=8

- (a) Given the matrix

$$A = \begin{pmatrix} 2 & 9 & 6 \\ 8 & 7 & 5 \\ 11 & 9 & 2 \end{pmatrix}$$

Suppose a row transformation has been

applied to it to obtain $B = \begin{pmatrix} 11 & 9 & 2 \\ 8 & 7 & 5 \\ 2 & 9 & 6 \end{pmatrix}$

(A and B are similar). Identify an appropriate elementary matrix such that multiplication of A by it would give B .

- (b) Given for a (3×3) matrix $|\text{adj}A| = 20$. Find $|A|$.

- (c) Show that if A and B are two unitary matrices, then their product AB will also be unitary.

- (d) Consider the system of non-homogeneous linear equations

$$(A)_{m \times n} (X)_{n \times 1} = (B)_{m \times 1}$$

It is given that the system is consistent and $\rho(A) = 5 \leq \min(m, n)$. Find the rank of the augmented matrix $(A:B)$, i.e., $\rho(A:B)$.

3. Answer any *three* of the following : $5 \times 3 = 15$

(a) State the three elementary operations on matrices. Show with an example, how an elementary row transformation can be implemented by pre-multiplication of the matrix with an appropriate elementary matrix.

(b) Show that for any two matrices A and B

$$\rho(AB) \leq \min(\rho(A), \rho(B))$$

(c) Compute the inverse of the following matrix by using elementary operations :

$$\begin{pmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix}$$

(d) Prove that the necessary and sufficient condition for a system of n homogeneous equations in n variables, i.e.,

$$(A)_{n \times n}(X)_{n \times 1} = (0)_{n \times 1}$$

to possess a non-trivial solution is that $|A|=0$ (i.e., $\det(A)=0$).

(e) Suppose x_1, x_2, \dots, x_n is a random sample of size n from a given population. Express the sample variance for this sample as a quadratic form. Identify a quadratic form in the multivariate normal distribution.

4. Answer any *three* of the following : $10 \times 3 = 30$

(a) Prove that the rank of a matrix A remains unchanged by pre-multiplication (or post-multiplication) with a non-singular matrix, i.e., $\rho(PA) = \rho(A) = \rho(AQ)$, where P and Q are non-singular matrices of appropriate order.

10

(b) What is meant by an orthogonal matrix? Show that if A and B are two n -rowed orthogonal matrices, then AB is also an orthogonal matrix. How is an orthogonal matrix different from a unitary matrix? Show that the following matrix

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

is unitary ($i^2 = -1$).

$2+4+2+2=10$

(c) (i) Suppose A is an $(m \times n)$ matrix and B is an $(n \times p)$ matrix and are partitioned as

$$A = \begin{pmatrix} R_1 \\ R_2 \\ \vdots \\ R_m \end{pmatrix}_{m \times n} \quad \text{and} \quad B = (C_1, C_2, \dots, C_p)_{n \times p}$$

where R_i is the i th row of A and C_j is the j th column of B ($i = 1, 2, \dots, m$ and $j = 1, 2, \dots, p$). Obtain AB in terms of R_i 's and C_j 's.

(6)

(ii) Given the matrix

$$M = \begin{pmatrix} 2 & 4 & 1 \\ 0 & 5 & 2 \\ 1 & 9 & 7 \\ 3 & -2 & 8 \end{pmatrix}$$

Obtain all possible minors of M .

$$6+4=10$$

(d) (i) Investigate for what values of λ and μ , the simultaneous equations

$$x + y + z = 6, \quad x + 2y + 3z = 10 \\ \text{and } x + 2y + \lambda z = \mu$$

have an infinite number of solutions.

(ii) Show that the following system of equations

$$x + y + z = -3, \quad 3x + y - 2z = -2 \\ \text{and } 2x + 4y + 7z = 7$$

are not consistent. $6+4=10$

(e) (i) Find the inverse of the matrix A and hence solve the equation $AX^T Y$, where

$$A = \begin{pmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{pmatrix}, \quad X = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \text{ and } Y = \begin{pmatrix} -1 \\ 1 \\ -5 \end{pmatrix}$$

(7)

(ii) Show that

$$B = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}$$

is the inverse of $A = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$.

$$6+4=10$$

(f) Consider the following quadratic form :

$$Q = 6X_1^2 + 3X_2^2 + 14X_3^2 \\ + 4X_2X_3 + 18X_3X_1 + 4X_1X_2$$

Find a transformation $Y = AX$, where

$$Y = \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix}, \quad X = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \text{ and } A \text{ is a } (3 \times 3)$$

matrix which reduces it to the form $6Y_1^2 - 29Y_2^2 + Y_3^2$. Also identify the index and signature of the quadratic form. 10
