3 (Sem–1) MAT M 1

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MATHEMATICS

(Major)

Paper : 1.1

(Algebra and Trigonometry)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×10=10

(a) What is the order of A_m, alternative group of degree n?

(b) Is generator of a cyclic group always unique?

(c) Does the set of all odd integers form a group with respect to addition?

(d) Define Hermitian matrix.

(e) What is normal form of a matrix?

(f) What is the rank of a matrix, where every element of the matrix is unity?

(g) If in a square matrix A, $|A| = 0$, then what is the value of $|\text{adj } A|$?

(h) Find the amplitude of the complex number $-1-i$.

(i) What is the period of $\sinh x$?

(j) State Gregory series.

2. Give the answer of the following questions : 2×5=10

(a) Can a non-Abelian group have an Abelian subgroup? Justify your answer.

(b) Express the following matrix as a sum of symmetric and skew-symmetric matrix :

$$A = \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$$

(c) Let A and B be two square matrices of order n. If $AB = 1$, then prove that $BA = 1$.

(d) If the matrices A and B commute, then show that A^{-1} and B^{-1} also commute.

(e) If

$$x_r = \cos\frac{\pi}{2^r} + i\sin\frac{\pi}{2^r}$$

then prove that $x_1 x_2 x_3 \cdots \infty = \cos\pi$.

3. Answer the following questions : 5×2=10

(a) Prove that every group of prime order is cyclic.

(b) Prove that i^i is completely real. Find its principal value.

Or

Prove that

$$\frac{1}{6}\sin^3 x = \frac{x^3}{\lfloor\underline{3}} - \frac{1}{\lfloor\underline{5}}(3^2+1)x^5 + \frac{1}{\lfloor\underline{7}}(3^4+3^2+1)x^7 + \cdots$$

4. Answer any *two* questions : 5×2=10

(a) If α, β, γ are the roots of the equation

$$x^3 - px^2 + qx - r = 0$$

then find the value of

$$\sum\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$$

in terms of p, q and r.

(b) Find the condition that the cubic

$$x^3 - px^2 + qx - r = 0$$

should have its roots in harmonic progression.

(c) Using Descartes' rule of sign, show that when n is even, the equation $x^n - 1 = 0$ has two real roots 1 and -1 and no other real root, and when n is odd, the only real root is 1.

5. Answer any *one* question : 10

(a) Let A be a non-empty set and let R be an equivalence relation in A. Let a and b be arbitrary elements in A. Then prove that—

(i) $[a] = [b]$, iff $(a, b) \in R$;

(ii) *either* $[a] = [b]$ *or* $[a] \cap [b] = \phi$.

(b) Prove that an equivalence relation R in a non-empty set S determines a partition of S and conversely, a partition of S defines an equivalence relation in S.

6. Answer any *one* question : 10

(a) If H is a subgroup of G, then prove that there is a one-to-one correspondence between the set of left cosets of H in G and the set of right coset of H in G.

(b) Prove that a subgroup H of a group G is a normal subgroup of G if and only if the product of two right cosets of H in G is again a right coset of H in G.

7. Answer any *one* question : 10

(a) Find real and imaginary parts of

$$\sin^{-1}(\cos\theta + i\sin\theta) \ (\theta \in R)$$

(b) If $\tan(\theta + i\phi) = \cos\alpha + i\sin\alpha$, prove that

$$\theta = \frac{n\pi}{2} + \frac{\pi}{4} \text{ and } \phi = \frac{1}{2}\log_e \tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$$

8. Answer any *one* question : 10

(a) If A be any n-square matrix, then show that

$$A(\text{Adj}A) = (\text{Adj}A)A = |A| I_n$$

where I_n is the n-rowed unit matrix. Verify it for the matrix

$$A = \begin{bmatrix} 2 & -1 \\ -3 & -2 \end{bmatrix}$$

(b) For what values of η, the equations

$$x+y+z=1$$
$$x+2y+4z=\eta$$
$$x+4y+10z=\eta^2$$

have a solution? Solve them completely in each case.

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