2016

MATHEMATICS

(Major)

Paper : 4.2

(Mechanics)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×10=10

(a) Write the conditions under which the moment of a system of coplanar forces about a point in their plane is zero.

(b) What is the length of arm of a couple equivalent to the couple (P, p) having constituent force of magnitude F?

(c) Write the least and the greatest value of the coefficient of friction.

(d) What is the position of the centre of gravity of a uniform circular ring?

(e) What is the amount of work done by a force causing displacement in a direction perpendicular to the line of action of the force?

(f) Classify the equilibrium of the following body as stable, unstable or neutral :

(i) A uniform solid sphere on a horizontal table

(ii) A uniform solid hemisphere in contact with its curved surface on a horizontal table

(g) What are null lines?

(h) If $P(r, \theta)$ is the position of a moving particle at any time t, then write the velocity component along the direction of $\bar{r}$ and along the increasing direction of θ.

(i) If the mass of a moving particle varies with time during its motion, what will be the appropriate equation in place of $P = m\dfrac{dv}{dt}$?

(j) When a particle falls under gravity in a medium whose resistance varies as the nth power of the velocity v, then write its maximum possible velocity.

2. Answer the following questions : 2×5=10

(a) Justify that if the moment of a system of coplanar forces about three points in their plane are different, then the resultant is a single force.

(b) State the laws of friction related to—

(i) the direction of the frictional force;

(ii) the magnitude of the limiting friction.

(c) If a_r and a_θ are the radial and cross-radial components of acceleration of a moving particle $P(r, \theta)$; then deduce its value if the particle moves along a circle of radius a.

(d) If a point moves in a straight line with SHM of amplitude a and has velocities v_1 and v_2 when its distances from the centre are x_1 and x_2 respectively, show that the period of oscillation is

$$2\pi\sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}$$

(e) State Kepler's laws of planetary motion.

3. Answer any *four* parts of the following : 5×4=20

(a) Show that the equation of the line of action of the resultant R of a system of coplanar forces acting at different points of a body is $xY - yX = M$, where X and Y are the resolved parts of R along x-axis and y-axis respectively and M is the sum of the moments of the forces about the origin.

(b) A uniform ladder is in equilibrium with one end resting on the ground and the other against a vertical wall. If the ground and the wall are rough, the coefficient of friction being μ and μ' respectively, and if the ladder is on the point of sliding at both ends and makes an angle θ with the horizontal, then show that

$$\tan\theta = \frac{1-\mu\mu'}{2\mu}$$

(c) Show that any system of forces acting on a rigid body can be reduced to a single force together with a couple whose axis is along the direction of the force. Hence define the Poinsot's Central Axis.

(d) Find the CG of an arc of a circle making an angle 2α at the centre. Hence, find the CG of a semicircular arc.

(e) A heavy uniform cube balances on the highest point of a sphere of radius r. If the sphere is rough enough to prevent sliding and if the side of the cube is $\frac{\pi r}{2}$, show that the cube can swing through a right angle without falling.

(f) A particle moves on a parabola $2a = r(1+\cos\theta)$ in such a manner that the component of the velocity at right angle to the radius vector from the focus is constant. Show that the acceleration of the point is constant in magnitude.

Answer any *four* questions from the following : 10×4=40

4. *(a)* Three forces P, Q, R act along the sides of a triangle formed by the lines $x+y=1$, $y-x=1$ and $y=2$. Find the equation of the line of action of the resultant. 5

(b) If D, E and F divide the sides BC, CA and AB respectively of an equilateral triangle ABC of sides a in the ratio 5 : 1, three forces each equal to P act at D, E and F perpendicularly to the sides and outwards from the triangle. Show that they are equivalent to a couple of moment Pa. 5

5. *(a)* Equal forces act along the axes and along the straight lines

$$\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$$

Find the equation of the central axis of the system. 5

(b) Three forces each equal to P act on a body; one at the point $(a, 0, 0)$ parallel to oy, the second at the point $(0, b, 0)$ parallel to oz and the third at the point $(0, 0, c)$ parallel to ox; the axes being rectangular, find the resultant wrench in magnitude and its position. 5

6. *(a)* A hemisphere rests in equilibrium on a sphere of equal radius. Show that the equilibrium is unstable or stable according as the curved surface or the flat surface of the hemisphere rests on the sphere. 6

(b) A heavy uniform rod of length $2a$, rests partly within and partly outside a fixed smooth horizontal bowl of radius r, the rim of the bowl is horizontal and one point of the rod is in contact with the rim; if θ is the inclination of the rod with the horizon, show that

$$2r\cos 2\theta = a\cos\theta$$ 4

7. *(a)* A particle is moving in a plane curve. Find the components of its acceleration along the tangent and the normal to the curve at any instant. 4

(b) If x is the distance covered by a particle of mass m at time t, falling under gravity in a medium whose resistance varies as the velocity, then show that

$$x = Vt-\left(\frac{V^2}{g}\right)\left(1-e^{-\left(\frac{gt}{V}\right)}\right)$$

V being the terminal velocity. 6

8. *(a)* Find the CG of the area bounded by the cardioid $r=a(1+\cos\theta)$. 5

(b) A particle moves in a straight line from a distance a towards the centre of force, the force varying inversely as the cube of the distance i.e., $\ddot{x}=-\frac{\mu}{x^3}$; show that velocity v at time t is given by

$$v^2=\mu\left(\frac{a^2-x^2}{a^2x^2}\right)$$ 5

9. *(a)* Deduce the differential equation of a central orbit in the polar form referred to the centre of force as the pole. 6

(b) If v_1 and v_2 are the velocities of a planet when it is respectively the nearest and the farthest from the sun, prove that

$$(1-e)v_1 = (1+e)v_2$$

e being the eccentricity of the ellipse. 4

10. *(a)* The velocities of a particle along and perpendicular to the radius vector from a fixed origin are λr and $\mu\theta$ respectively. Find the path of the particle and show that the acceleration components along and perpendicular to the radius vector are $\lambda^2 r - \frac{\mu^2\theta^2}{r}$ and $\mu\theta\left(\lambda + \frac{\mu}{r}\right)$. 5

(b) A particle is projected along the inner surface of a rough sphere and is acted on by no force. Show that it will return to the point of projection after time

$$\frac{a}{\mu V}(e^{2\pi\mu} - 1)$$

where a is the radius of the sphere, V is the velocity of projection and μ is the coefficient of friction. 5

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