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3 (Sem-1/CBCS) PHY HC 1

2022

PHYSICS

(Honours)

Paper : PHY-HC-1016

(Mathematical Physics-I)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer **any seven** of the following questions: 1×7=7

(a) Define unit vectors.

(b) If $\vec{A} \cdot \vec{B} = 0$, then what is the angle between $\vec{A}$ and $\vec{B}$?

(c) What is a 'DEL' operator ?

(d) Find the Laplacian of the scalar field

$$\phi = x y^2 z^3$$

Contd.

- (e) State Green's theorem.
- (f) Write the order and degree of the differential equation

$$2y \left(\frac{d^2 y}{dx^2} \right) + \left(\frac{dy}{dx} \right)^4 = 0$$

- (g) What do you understand by the statement $\vec{\nabla} \cdot \vec{A} = 0$?
- (h) What is an 'error' in statistics?
- (i) Define coordinate surfaces in curvilinear co-ordinates.
- (j) Write the integrating factor of the differential equation

$$\frac{dy}{dx} + 5y = x^2$$

- (k) Write the geometrical interpretation of the scalar triple product.
- (l) Define variance in statistics.

2. Answer **any four** of the following questions :
2×4=8

- (a) Give examples of a scalar field and a vector field.

- (b) If $\vec{r}$ represents the position vector, then find the value of $\vec{\nabla} \cdot \vec{r}$.

- (c) Define the line integral of a vector.
- (d) Write down the relation of cylindrical co-ordinate (r, θ, z) with cartesian co-ordinate (x, y, z) .
- (e) Explain the scale factors h_1, h_2, h_3 in curvilinear co-ordinate system.
- (f) For what value of N , the vectors $\vec{A} = 2\hat{i} + 3\hat{j} - 6\hat{k}$ and $\vec{B} = N\hat{i} + 2\hat{j} + 2\hat{k}$ are perpendicular to each other.

- (g) Evaluate $\iint_S \vec{r} \cdot \hat{n} ds$, where S is a closed surface.

- (h) Prove that $\delta(x) = \delta(-x)$.

3. Answer **any three** of the following questions :
5×3=15

- (a) Show that

$$\frac{d}{dt} (\vec{A} \times \vec{B}) = \frac{d\vec{A}}{dt} \times \vec{B} + \vec{A} \times \frac{d\vec{B}}{dt}$$

(b) If $\phi = xy + yz + zx$ and $\vec{F} = \vec{\nabla} \phi$, then find $\vec{\nabla} \cdot \vec{F}$ and $\vec{\nabla} \times \vec{F}$.

(c) Apply Green's theorem in the plane to evaluate the integral

$$\oint_C [(xy - x^2)dx + x^2y dy]$$

over the triangle bounded by the lines $y = 0$, $x = 1$ and $y = x$.

(d) Solve the differential equation

$$2xy \frac{dy}{dx} = x^2 + 3y^2$$

(e) Express $\nabla^2 \psi$ in cylindrical coordinate system.

(f) Prove that

$$\delta(x^2 - a^2) = \frac{1}{2a} [\delta(x - a) + \delta(x + a)]$$

(g) A function is defined as

$$f(x) = \begin{cases} 0 & \text{for } x < 2 \\ \frac{1}{18}(2x + 3) & \text{for } 2 \leq x \leq 4 \\ 0 & \text{for } x > 4 \end{cases}$$

Show that it is a probability density function.

(h) If $\vec{F}$ is a vector, prove that

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{F}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{F}) - \nabla^2 \vec{F}$$

4. Answer **any three** of the following questions: 10×3=30

(a) (i) Show that the gradient of a scalar field is a vector. 5

(ii) Show that 2½×2=5

1. $\text{div curl } \vec{A} = 0$ and

2. $\text{curl}(\text{grad } \phi) = 0$

(b) (i) Define curvilinear co-ordinate system. When it is called orthogonal? 3+1=4

(ii) Obtain expression for length, area and volume elements in curvilinear coordinate system. 2+2+2=6

(c) (i) State and explain Gauss-divergence theorem. 3

(ii) Give the physical meaning of divergence and curl of a vector. 2+2=4

(iii) Find an expression of $\vec{\nabla} \cdot \vec{A}$ in spherical polar co-ordinate system. 3

- (d) (i) Find the directional derivative of $\phi(x, y, z) = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of vector $\hat{i} + 2\hat{j} + 2\hat{k}$. 5

(ii) Prove that $\nabla^2 \left(\frac{1}{r} \right) = 0$. 5

- (e) Solve the following differential equations: 5+5=10

(i) $(1 + x^2) \frac{dy}{dx} + 2xy = \cos x$

(ii) $\frac{d^2y}{dx^2} - 5 \frac{dy}{dx} + 6y = e^{4x}$

- (f) State and prove Stoke's theorem. Using Stoke's theorem show that

$$\oint_C \vec{r} \times d\vec{r} = 2 \iint_S d\vec{S}, \text{ where } C \text{ is the closed}$$

perimeter curve bounding the open surface S . 1+5+4=10

- (g) (i) Solve $\frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + 2y = 0$, subject to the condition $y(0) = 0, y'(0) = 1$

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- (ii) Prove that

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = (\vec{A} \times \vec{B}) \cdot \vec{C} \quad 4$$

- (h) (i) If $\vec{A} = 6\hat{i} + 4\hat{j} + 3\hat{k}$
 $\vec{B} = 2\hat{i} - 3\hat{j} - 3\hat{k}$
 $\vec{C} = \hat{i} + \hat{j} + \hat{k}$ then evaluate

$$\vec{A} \times (\vec{B} \times \vec{C}) \quad 4$$

- (ii) Evaluate $\oint_C x^2 y dx + y^2 dy$, where C is the boundary of the region enclosed by $y = x$ and $y^2 = x$. 6